

A METHOD OF RADAR PATTERN RECOGNITION BY SORTING  
SIGNALS INTO DATA CLUSTERS

TECHNICAL FIELD

The present invention relates, in general, to radar identification and, more specifically, to sorting signals received from multiple emitters into data clusters for pattern recognition.

5

BACKGROUND OF THE INVENTION

Radars emit a variety of signals that may characterize and identify them. Each radar may emit a specific pulse amplitude and a specific fixed radio frequency (RF) or a variable RF ranging over a fixed bandwidth. Each may emit a fixed pulse repetition frequency (PRF) or pulse repetition interval (PRI) and may be  
10 of a certain pulse width (PW).

An aircraft flying into a region with an onboard wideband receiver may detect a variety of signals emitted from multiple radars located in that region. Unless these signals are sorted and separated from each other, it is not possible for the aircraft to determine the types of classes of radars it is about to encounter. It

does not know whether the radars are hostile and does not know whether the radars present a high or low threat to the incoming aircraft.

A need, therefore, exists for an aircraft to be able to sort and identify the variety of radars that are emitting energy towards the aircraft. The present  
5 invention addresses this need.

### SUMMARY OF THE INVENTION

To meet this and other needs, and in view of its purposes, the present invention provides a method of classifying radar emitters including the steps of: (a) receiving a plurality of signals from the radar emitters; (b) generating data  
10 components for each signal received from the radar emitters; (c) forming multi-dimensional samples using the generated data components; and (d) sorting the multi-dimensional samples into a plurality of data clusters, based on their respective proximity to the data clusters, each data cluster representing a classification of a radar emitter. Step (b) includes generating pulse data descriptors (PDWs) during a  
15 predetermined interval of time, and generating at least radio frequency (RF) data and pulse width (PW) data for the PDWs.

In another embodiment, the invention provides a system for classifying radar emitters including a receiver for receiving a plurality of signals from the radar emitters, and a processor coupled to the receiver, for (a) generating data  
20 components for each signal received from the radar emitters, (b) forming multi-dimensional samples from the generated data components; and (c) sorting the multi-dimensional samples into a plurality of data clusters, based on their respective

proximity to the data clusters, each data cluster representing a classification of a radar emitter.

The processor may also assign a multi-dimensional sample to a data cluster, based on a Euclidean distance between the multi-dimensional sample and a center of the data cluster. The center of the data cluster may be formed as a mean vector of a set of multi-dimensional samples assigned to the data cluster.

In yet another embodiment, the invention provides a machine readable storage medium containing a set of instructions for a computer. The set of instructions implements the following steps: (a) processing a plurality of signals received from a receiver; (b) generating data components for each signal received from the receiver; (c) forming multi-dimensional samples using the generated data components; and (d) sorting the multi-dimensional samples into a plurality of data clusters, based on their respective proximity to the data clusters, each data cluster representing a classification of a radar emitter. Step (d) may include sorting the multi-dimensional samples using an ISODATA (iterative self-organizing data analysis technique) computer algorithm.

It is understood that the foregoing general description and the following detailed description are exemplary, but are not restrictive, of the invention.

BRIEF DESCRIPTION OF THE DRAWING

The invention is best understood from the following detailed description when read in connection with the accompanying drawing. Included in the drawing are the following figures:

5                   FIG. 1 is a flow diagram showing steps performed by a conventional ISODATA computer algorithm;

                  FIG. 2 is a system block diagram illustrating a radar threat sorting, clustering and classification system, in accordance with an embodiment of the present invention;

10                  FIG. 3 is an example of an input snapshot showing radio frequency (RF) versus time of arrival, as received by a wideband receiver included in an embodiment of the present invention;

                  FIG. 4 is an example of another input snapshot showing pulse width versus time of arrival, as received by a wideband receiver included in an embodiment  
15                  of the present invention;

                  FIG. 5 is yet another example of an input snapshot showing pulse amplitude versus time of arrival, as received by a wideband receiver included in an embodiment of the present invention;

FIG. 6 is still another input snapshot showing RF frequency versus pulse width, as received by a wideband receiver included in an embodiment of the present invention;

FIG. 7 is a plot of RF frequency versus pulse width resulting after one iteration of clustering, as performed in accordance with an embodiment of the present invention;

FIG. 8 is a plot of RF frequency versus pulse width resulting after four iterations of clustering, as performed by an embodiment of the present invention;

FIG. 9 is a plot of RF frequency versus pulse width resulting after six iterations of clustering, as performed by an embodiment of the present invention; and

FIG. 10 is a plot of RF frequency versus pulse width resulting after seven iterations of clustering, as performed by an embodiment of the present invention.

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#### DETAILED DESCRIPTION OF THE INVENTION

The present invention provides an unsupervised iterative classification method for radar pattern recognition. The method is self-organizing and requires minimal input from human interaction.

The method of the invention forms clusters from a set of input data (samples), where each cluster consists of very similar data (samples). The method

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first defines a measure of pattern similarity and establishes a rule for assigning individual samples to the domain of a specific cluster center. The invention uses the Euclidean distance between two data points  $x$  and  $z$ ,

$$D = ||x - z||$$

- 5 as a measure of pattern similarity. The smaller the distance,  $D$ , the greater is the similarity between  $x$  and  $z$ .

After a measure of pattern similarity is selected, the method of the invention sorts (or partitions) samples into cluster domains. The Euclidean distance measure,  $D$ , lends itself to this procedure, because it is a good measure of proximity.  
10 However, because the proximity of two patterns is a relative measure of similarity, it is necessary for the invention to establish a threshold to define degrees of acceptable similarity for the clustering method.

A performance-index is chosen to measure the degrees of similarity and a procedure is used which minimizes the chosen performance index. One such  
15 performance index is the sum of the squared errors resulting from the cluster, and is a proximity measure given by

$$J = \sum_{j=1}^{N_c} \sum_{x \in S_j} ||x - m_j||^2$$

where  $N_c$  is the number of cluster domains (or simply clusters),  $S_j$  is the set of samples belonging to the  $j$ th domain, and

$$m_j = (1/N_j) \sum_{x \in S_j} x$$

is the sample mean vector of set  $S_j$ , with  $N_j$  representing the sample size of  $S_j$ .

There are other performance indices used in the method of clustering the samples, such as: (1) average squared distances between samples in a cluster domain, (2) average squared distances between samples in different cluster domains, (3) indices based on a scatter matrix and (4) minimum and maximum variance indices.

An embodiment of the invention will now be described based on an algorithm referred to as Iterative Self Organizing Data Analysis Techniques (ISODATA), which is disclosed by J. T. Tou and R. C. Gonzalez, Pattern Recognition Principles, Addison-Wesley, 1974, Chapter 3, pp. 75-109. The ISODATA algorithm, generally designated as 10, is also shown in FIG. 1, and is further described below.

For a set of  $N$  samples,  $\{x_1, x_2, \dots, x_N\}$ , ISODATA clustering algorithm includes the following steps:

Step 1: Specify various clustering parameters, as follows:

$K$  = number of cluster centers desired;

$\theta_N$  = the minimum number of samples allowed in a cluster;

$\theta_S$  = standard deviation parameter;

$\theta_c$  = lumping parameter;

$L$  = maximum number of pairs of cluster centers which may be lumped;

$I$  = number of iterations allowed.

Step 2: Distribute the  $N$  samples among the present cluster centers,  
5 using the following relationship:

$$x \in S_j \text{ if } \|x - z_j\| \leq \|x - z_i\|, \quad i = 1, 2, \dots, N_c; \quad i \neq j$$

for all  $x$  in the sample set. In this notation,  $S_j$  represents the subset of samples assigned to the cluster center  $z_j$ .

Step 3: Discard sample sets with fewer than  $\theta_N$  members. That is, if  
10 for any  $j$ ,  $N_j < \theta_N$ , discard  $S_j$  and reduce  $N_c$  by 1.

Step 4: Update each cluster center  $z_j$ ,  $j = 1, 2, \dots, N_c$ , by setting it equal to the sample mean of its members ( $S_j$ ), as follows:

$$z_j = \frac{1}{N_j} \sum_{x \in S_j} x, \quad j = 1, 2, \dots, N_c$$

where  $N_j$  is the number of samples in  $S_j$ .

15 Step 5: Compute the average distance  $D_j$  of samples in cluster domain  $S_j$  from their corresponding cluster center, using the following relationship:



$$D_j = \frac{1}{N_j} \sum_{x \in S_j} \|x - z_j\|, \quad j = 1, 2, \dots, N_c$$

Step 6: Compute the overall average distance of the samples from their respective cluster centers, using the following relationship:

$$D_{ave} = \frac{1}{N_c} \sum_{j=1}^{N_c} N_j D_j$$

5

Step 7: The following decisions are then made:

(a) If this is the last iteration, set  $\theta_c = 0$  and go to Step 11;

(b) If  $N_c \leq K/2$ , then go to Step 8;

(c) If this is an even-numbered iteration, or if  $N_c \geq 2K$ , go to Step 11; otherwise continue.

10

Step 8: Find the standard deviation vector  $\sigma_j = (\sigma_{1j}, \sigma_{2j}, \dots, \sigma_{nj})'$  for each sample subset, using the following relationship:

$$\sigma_{ij} = \sqrt{\frac{1}{N_j} \sum_{x \in S_j} (x_{ik} - z_{ij})^2}, \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, N_c$$

15

where  $n$  is the sample dimensionality,  $x_{ik}$  is the  $i$ th component of the  $k$ th sample in  $S_j$ ;  $z_{ij}$  is the  $i$ th component of  $z_j$ , and  $N_j$  is the number of sample in  $S_j$ . Each component of  $\sigma_j$  represents the standard deviation of the samples in  $S_j$  along a principal coordinate axis.

Step 9: Find the maximum component of each  $\sigma_j$ ,  $j = 1, 2, \dots, N_c$  and denote it by  $\sigma_{j\max}$ .

Step 10: If for any  $\sigma_{j\max}$ ,  $j = 1, 2, \dots, N_c$ , there are  $\sigma_{j\max} > \theta_s$  and

$$(a) \quad D_j > D_{ave} \text{ and } N_j > 2(\theta_N + 1),$$

5

or

$$(b) \quad N_c \leq K/2$$

then split  $z_j$  into two new cluster centers  $z_j^+$  and  $z_j^-$ , delete  $z_j$ , and increase  $N_c$  by 1.

Cluster center  $z_j^+$  is formed by adding a given quantity  $\gamma_j$  to the component  $z_j$  which corresponds to the maximum component of  $\sigma_j$ , ( $\sigma_{j\max}$ ). Similarly,  
10  $z_j^-$  is formed by subtracting  $\gamma_j$  from the same component of  $z_j$ . One way of specifying  $\gamma_j$  is to let it be equal to a fraction of  $\sigma_{j\max}$ , that is  $\gamma_j = k\sigma_{j\max}$  with  $0 < k < 1$ .

If splitting took place in this step, then go to Step 2; otherwise continue.

Step 11: Compute the pairwise distances  $D_{ij}$  between all cluster  
15 centers, as follows:

$$D_{ij} = \|z_i - z_j\|, \quad i=1, 2, \dots, N_c-1; j=i+1, \dots, N_c$$

Step 12: Compare the distances  $D_{ij}$  against the parameter  $\theta_c$ .

Arrange the  $L$  smallest distances which are less than  $\theta_c$  in ascending order, as follows:

$$[ D_{i_1j_1}, D_{i_2j_2}, \dots, D_{i_Lj_L} ]$$

- 5 where  $D_{i_1j_1} < D_{i_2j_2} < \dots < D_{i_Lj_L}$  and  $L$  is the maximum number of pairs of cluster centers which may be lumped together. The lumping process is described below in Step 13.

Step 13: With each distance  $D_{i_kj_k}$ , there is associated a pair of cluster centers  $z_{i_k}$  and  $z_{j_k}$ . Starting with the smallest of these distances, perform a pairwise lumping operation, according to the following relationship:

- 10 For  $k = 1, 2, \dots, L$ , if neither  $z_{i_k}$  nor  $z_{j_k}$  has been used in lumping during this iteration, merge these two cluster centers, using the following relationship:

$$z_k^* = \frac{1}{N_{i_k} + N_{j_k}} [ N_{i_k} (z_{i_k}) + N_{j_k} (z_{j_k}) ]$$

Delete  $z_{i_k}$  and  $z_{j_k}$  and reduce  $N_c$  by 1.

- 15 It is noted that only pairwise lumping is allowed and that a lumped cluster center may be obtained by weighting each old cluster by the number of samples in its domain. It will be understood that since a cluster center can only be lumped once, this step may not always result in  $L$  lumped centers.

Step 14: If this is the last iteration, the algorithm terminates. Otherwise, go to Step 1 if any of the process parameters requires changing at the user' s discretion, or go to Step 2 if the parameters are to remain the same for the next iteration. An iteration is counted every time the procedure returns to Step 1 or 2.

Based on a flowchart of the ISODATA algorithm, illustrated in FIG. 1, the inventors developed a computer program, as described below, for radar threat clustering and radar identification/recognition based on the clustering. The program was written in MATLAB, although other languages may have been used.

A listing of the MATLAB program for clustering radar data samples is provided in the following tables.

Table A, threat\_gen\_n.m, lists a program for generating a snapshot of the radars' pulse descriptive words (PDWs). The snapshot includes PDW mixes from multiple radar threats, as they may be intercepted by wideband receiver 21, as shown in FIG. 2. Exemplary snapshots (80 ms duration) are shown in FIGS. 3-6 (discussed later).

Table A. Program threat\_gen\_n.m

```

clear all;
colors=['r','b','k','g','m','y','p'];
% -----
5 %
% --- Program threat_gen_n.m
%
% This program is used to generate threat signal PDW for isodata.m
%
10 % -----
%
ss="";
%
% argument for threatlist of each row:
15 %
% freq,std_freq,aoa,std_aoa,pw,std_pw,pa,std_pa,toa,std_toa,pri,id
%
%
threatlist=[900,200,35,10,8,2,-5,10,22,0,1120,1;...
20           1050,100,40,10,14,2,-15,10,300,0,1820,2;...
           1150,150,45,10,4,2,-20,10,1700,0,1920,3;...
           1310,190,50,10,6.5,2,-35,10,700,0,1320,4;...
           1210,5,53,4,12,2,-45,10,1900,0,2920,5];
%
25 %the weight set: w1=0.; w2=0.1; w3=0.1; w4=0.1; w5=0.1; w6=0.;
%
%

30 [row,col]=size(threatlist);
    for threatnum=1:row
        a=threat(threatlist(threatnum,:));
        tmptxt=sprintf('threat_%d=a;',threatnum); %dynamically name and create
variable
35         eval(tmptxt);
        ss=cat(1,ss,a);
    end
ss_sorted=sortrows(ss,1);
sss=zeros(size(ss));
40 sss=ss_sorted;
[row,col]=size(sss);
%
% -----
%
45 % plotting input PDWs
%
% ss_sorted(:,1) - TOA
% ss_sorted(:,2) - AOA

```

```

% ss_sorted(:,3) - RF Freq
% ss_sorted(:,4) - Pulsewidth
% ss_sorted(:,5) - Pulse Amp
% ss_sorted(:,6) - Threat ID
5 % -----
%
figure
scatter(ss_sorted(:,4),ss_sorted(:,3),5,ss_sorted(:,6),'filled')
title(' Input Threat PDWs in 80 ms Snapshot','FontSize',12,'FontWeight','bold')
10 xlabel('Pulsewidth (usec)');
ylabel('RF Frequency (MHz)');
%
figure
scatter(ss_sorted(:,1),ss_sorted(:,2),5,ss_sorted(:,6),'filled')
15 title(' Input Threat PDWs in 80 ms Snapshot','FontSize',12,'FontWeight','bold')
xlabel('Time of Arrival (usec)');
ylabel('Angle of Arrival (deg)');
%
figure
20 scatter(ss_sorted(:,1),ss_sorted(:,3),5,ss_sorted(:,6),'filled')
title(' Input Threat PDWs in 80 ms Snapshot','FontSize',12,'FontWeight','bold')
xlabel('Time of Arrival (usec)');
ylabel('RF Frequency (deg)');
%
25 figure
scatter(ss_sorted(:,1),ss_sorted(:,4),5,ss_sorted(:,6),'filled')
title(' Input Threat PDWs in 80 ms Snapshot','FontSize',12,'FontWeight','bold')
xlabel('Time of Arrival (usec)');
ylabel('Pulsewidth (usec)');
30 %
figure
scatter(ss_sorted(:,1),ss_sorted(:,5),5,ss_sorted(:,6),'filled')
title(' Input Threat PDWs in 80 ms Snapshot','FontSize',12,'FontWeight','bold')
35 xlabel('Time of Arrival (usec)');
ylabel('Pulse Amplitude (dBm)');
%
for colindex=2:(col-1)
    sigma=std(sss(:,colindex));
    mean1=mean(sss(:,colindex));
40    sss(:,colindex)=(sss(:,colindex)-mean1)./sigma*5;
end
%
mean_value=[mean(ss(:,2)),mean(ss(:,3)),mean(ss(:,4)),mean(ss(:,5))];
std_value=[std(ss(:,2)),std(ss(:,3)),std(ss(:,4)),std(ss(:,5))];
45 n_total=length(sss);
%
disp('***** Input PDWs *****')
disp(ss_sorted)
disp('***** Normalized PDWs *****')
50 disp(sss)
disp('***** Mean Value Vector *****')

```

```

disp(mean_value)
disp('***** STD Vector *****')
disp(std_value)
disp('***** Sample Size *****')
5  disp(n_total)
   %
   % store generated PDW's
   %
   dlmwrite('test_pdw_n.out',sss)
10  dlmwrite('mean_value_n.out',mean_value)
   dlmwrite('std_value_n.out',std_value)
   dlmwrite('n_total_n.out',n_total)
   %
   %
15  % --- end of program

```

Each PDW, which is a vector, is composed of four components, describing an intercepted radar pulse, as follows: (1) time of intercept (or arrival), TOA, (2) radio frequency, RF, (3) pulse width, PW, and (4) pulse amplitude, PA. It will be appreciated that in other embodiments of the present invention, less or more than four components (dimensions) of each PDW may be selected. For example, other components may be pulse repetition interval (PRI), modulation type, etc.

Referring to FIG. 2, system 20 includes wideband receiver 21 for receiving desired components of each radar 1-N. Also included is processor 26, coupled to wideband receiver 21, for generating each PDW using PDW generator 22, normalizing each PDW using PDW normalizer 23 and clustering each normalized PDW into a respective cluster using ISODATA module 24. The ISODATA module executes the steps of the ISODATA algorithm. After a predetermined number of iterations of the ISODATA algorithm, the clusters of PDWs may be formed and provided to radar classifier module 25, which may be included in processor 26 or may be a separate module. By matching the clusters against stored table 27, the latter containing

identifications of known radars (threats and/or non-threats), the radars may be classified and identified.

Each raw PDW is normalized by module 23 of system 20, using the following relationship:

$$5 \quad PDW_{Nor} = [PDW_{Raw} - PDW_{Ave}] / STD_{PDW}$$

where  $PDW_{Nor}$  is the individual normalized PDW vector,  $PDW_{raw}$  is the individual PDW as intercepted by wideband receiver 21,  $PDW_{Ave}$  is the average PDW vector of the entire snapshot, and  $STD_{PDW}$  is the standard deviation vector calculated from  $PDW_{raw}$  and  $PDW_{Ave}$ .

10                      Table B, threat.m, lists a MATLAB function called by threat\_gen\_n.m to generate the PDWs.



Table B. Program threat.m

```

function a=threat(inputvector)
% -----
%
5  % --- Program threat.m
%
% This program is used to generate PDW for threat
%
% -----
10 %
% Snapshot Size
%
nt=80000;
%
15 % parameters of each threat
%
freq=inputvector(1);
std_freq=inputvector(2);
aoa=inputvector(3);
20 std_aoa=inputvector(4);
pw=inputvector(5);
std_pw=inputvector(6);
pa=inputvector(7);
std_pa=inputvector(8);
25 toa=inputvector(9);
std_toa=inputvector(10);
pri=inputvector(11);
ww=inputvector(12);
%
30 % generate threat PDW's with n types each with k samples
%
np2=floor((nt-toa(1))/pri);
%disp(np2)
%
35 a1=toa(1);
for irun=1:np2
    a1=a1+pri;
    a2=aoa(1)+(rand(1)-0.5)*std_aoa(1);
    a3=freq(1)+(rand(1)-0.5)*std_freq(1);
40    a4=pw(1)+(rand(1)-0.5)*std_pw(1);
    a5=pa(1)+(rand(1)-0.5)*std_pa(1);
    a6=ww(1);
    a(irun,:)= [a1 a2 a3 a4 a5 a6];
end

```

Table 1, isodata\_n.m, is the main program, which executes Step 1, Step 7 and Step 14 of the ISODATA algorithm, executed by module 24 of FIG. 2. In the exemplary embodiment of the invention, the performance index for measuring similarity between two PDWs uses only two components, namely RF and PW.

5                   The Euclidean distance,  $D_{ij}$ , between PDWs ( $PDW_i$  and  $PDW_j$ ) is calculated as follows:

$$D_{ij} = w_1(RF_i - RF_j)^2 + w_2(PW_i - PW_j)^2$$

where ( $RF_i$ ,  $PW_i$ ) and ( $RF_j$ ,  $PW_j$ ) represent  $PDW_i$  and  $PDW_j$ , respectively. Two weights,  $w_1$  and  $w_2$  are used, as an example, to adjust the relative size of the cluster (or  
10                   equivalently, the pairwise distance between cluster centers) to be generated in ISODATA. The relative size may be adjusted as a function of the overall frequency and pulse width deviations, which likely are related to the number of input radar threats of the input snapshot, or may be adjusted as a function of dedicated frequency bands in which advanced emitters may reside and need to be clustered  
15                   into a specific cluster size.

Referring to Table 1, six weights are listed ( $w_1 - w_6$ ). All weights are set to zero, except  $w_3$  and  $w_4$ , which are RF frequency and pulse width, respectively. It will also be appreciated that initially at the start of the ISODATA algorithm, the number of clusters may be assumed to be 1. Samples too far away from a center of  
20                   this original cluster may then be dropped from the cluster and a new cluster may be formed from the dropped samples.

Table 1. Program ISODATA n.m

```

%           Program isodata_n.m
%
% Interactive Self-Organizing Data Analysis Technique Algorithm
5  %           (ISODATA)
%
% input Data to be clustered
%
%
10 ss=dlmread('test_pdw_n.out');      % input pdws
   fmean=dlmread('mean_value_n.out'); % mean value vector
   fstd=dlmread('std_value_n.out');    % std value vector
   n_total=dlmread('n_total_n.out');  % total number of input pdws
%
15 %
   np=n_total;
   ndim=6;
%
% parameters and initial conditions
20 %
   w1=0.;    %TOA
   w2=0.0;   %AOA
   w3=0.1;   %RF Freq
   w4=0.15;  %Pulsewidth
25 w5=0.;    %Pulse Amp
   w6=0.;    %ID
%
% initial number of cluster = 1
%
30 nc=1;
%
   z(1,:)=ss(1,:);
%
% -----
35 % ***** Step 1 (specify parameters) *****
% -----
%
   k=18;      % number of cluster centers desired
   theta_n=1; % min numbers allowed in a cluster
40 %
% -----
%
% original settings:
%
45 %theta_s=0.75; % std parameter
   %theta_c=2;   % lumping parameter
%
%
   theta_s=0.75; % std parameter
50 theta_c=1;    % lumping parameter

```

```

%
% -----
%
5  L=0;          % max number of pairs of cluster centers allowed to be lumped
   Imax=12;      % number of iterations allowed
%
%
10 disp('***** SODATA Cluster Seeking Algorithm *****')
   %
   disp('***** Number of Data Points *****')
   disp(np)
   %
   disp('***** Dimension of PDW *****')
15  disp(ndim)
   %
   disp('***** Input Normalized PDWs Data *****')
   disp(ss)
   disp('***** Input Mean Value Vector *****')
20  disp(fmean)
   disp('***** Input STD Vector *****')
   disp(fstd)
   %
   disp('***** Number of Clusters Desired *****')
25  disp(k)
   %
   disp('***** Min Number of Data Required in a Cluster *****')
   disp(theta_n)
   %
30  disp('***** STD Parameter *****')
   disp(theta_s)
   %
   disp('***** Lumping Parameter *****')
   disp(theta_c)
35  %
   disp('***** Max Number of Cluster-Pairs Allowed to be Lumped *****')
   disp(L)
   %
   disp('***** Max Number of Iterations Allowed *****')
40  disp(Imax)
   disp(' ')
   disp('***** Start Isodata Algorithm *****')
   disp('*****')
   %
45  %
   for irun = 1:Imax
       % number of total "do loops" to step 14
       %
       % set initial condition for each loop -----
       %
50  goto2=0;
      goto11=0;

```

```

split=0;
%
% -----
%
5   step2
    step3
    step4
    step5
    step6
10  %
    % -----
    % ***** step 7 *****
    % -----
    %
15  if (irun == Imax)
        theta_c=0;
        goto11=1;
    end
    %
20  if (irun ~= Imax & (mod(irun,2) == 0 | nc >= 2*k))
        goto11=1;
    end
    %
    if (irun ~= Imax & nc > k/2)
25      goto11=1;
    end
    %
    if goto11 ~= 1
        step8
30      step9
        step10
        nc=ncc;
        if split == 1
            goto2 =1;
35      end
    end
    %
    if goto2 ~=1
        step11
40      step12
    end
end
                                     %main loop
%
% Summary of Clusters' Positions
45 %
disp(' ')
disp('***** Summary of Clustering Results')
disp('*****')
disp(' ')
50 disp('***** Clusters Positions *****')
sc=5;

```

```
for ipr=1:nc
    z(ipr,2)=z(ipr,2)*fstd(1)/sc+fmean(1);
    z(ipr,3)=z(ipr,3)*fstd(2)/sc+fmean(2);
    z(ipr,4)=z(ipr,4)*fstd(3)/sc+fmean(3);
5    z(ipr,5)=z(ipr,5)*fstd(4)/sc+fmean(4);
end
disp(z)
disp('***** Clusters Members *****')
for ipr=1:np
10    yy(ipr,2)=yy(ipr,2)*fstd(1)/sc+fmean(1);
    yy(ipr,3)=yy(ipr,3)*fstd(2)/sc+fmean(2);
    yy(ipr,4)=yy(ipr,4)*fstd(3)/sc+fmean(3);
    yy(ipr,5)=yy(ipr,5)*fstd(4)/sc+fmean(4);
end
15 disp(yy)
```

Table, 2, step2.m, lists the program to perform ISODATA Step 2.

Table 2. Program STEP2.m

```

% ***** Step 2 (cluster data based on Euclidean distances) *****
% -----
5 disp('***** Step 2 *****')
disp('*** Run Number ***')
disp(irun)
%
% If there are more than 1 clusters
10 %
disp('*** Number of Clusters *****')
disp(nc)
    if nc > 1
        % for multiple clusters,
% the else at line 94
15 %
% Euclidean Distance
%
    for idis=1:nc
        for kdis=1:np
20            dis2=w1*(ss(kdis,1)-z(idis,1))^2 + w2*(ss(kdis,2)-z(idis,2))^2 + ...
                w3*(ss(kdis,3)-z(idis,3))^2 + w4*(ss(kdis,4)-z(idis,4))^2 + ...
                w5*(ss(kdis,5)-z(idis,5))^2 + w6*(ss(kdis,6)-z(idis,6))^2;
            dis(kdis,idis)=sqrt(dis2);
        end
25 end
% for kdis at line 17
% for idis at line 16
%
% Sorting into clisest cluster center
%
    for kdis=1:np
30        comp=dis(kdis,1);
        i_sort(kdis)=1;
        for idis=1:nc
            if dis(kdis,idis) < comp
                comp = dis(kdis,idis);
35                i_sort(kdis)=idis;
            end
            % for if at line 31
        end
        % for idis at line 30
    end
    % for kdis at line 27
40 %
% Count how many members in each cluster
%
    for idis=1:nc
        n_total(idis)=0;
        for kdis=1:np
45            if i_sort(kdis) == idis
                n_total(idis)=n_total(idis)+1;
            end
            % for if at line 43
        end
    end
end

```

```

        end                                % for kdis at line 42
    end                                    % for idis at line 40
    %
    % If there is only one cluster
5    %
    else                                % for if at line 12, only one cluster
        for kdis=1:np
            i_sort(kdis)=1;
        end                                % for kids at line 52
10    n_total(1)=np;
    end                                % for else at line 51
    %
    % print present cluster centers
    %
15    disp('***** Present Cluster Centers *****')
    %
    for iprint=1:nc
        z(iprint,:);
    end                                % for iprint at line 62
20    disp(z)
    %
    for iprint=1:np
        yy(iprint,:)=[ss(iprint,:) i_sort(iprint)];
    end
25    disp(' ***** Present Cluster Members *****')
    disp('                Input Data                Cluster')
    disp(yy)
    sc=5;
    for ipr=1:np
30    yz(ipr,1)=yy(ipr,1);
        yz(ipr,2)=yy(ipr,2)*fstd(1)/sc+fmean(1);
        yz(ipr,3)=yy(ipr,3)*fstd(2)/sc+fmean(2);
        yz(ipr,4)=yy(ipr,4)*fstd(3)/sc+fmean(3);
        yz(ipr,5)=yy(ipr,5)*fstd(4)/sc+fmean(4);
35    yz(ipr,6)=yy(ipr,6);
        yz(ipr,7)=yy(ipr,7);
    end
    %
    % plot clustering results after each run
40    %
    % -----
    %
    % yz(:,1) - TOA
    % yz(:,2) - AOA
45    % yz(:,3) - RF Freq
    % yz(:,4) - Pulsewidth
    % yz(:,5) - Pulse Amp
    % yz(:,6) - Threat ID
    % yz(:,7) - Cluster Number
50    %
    % -----

```



```

%
figure
scatter(yz(:,4),yz(:,3),5,yz(:,7),'filled')
title(sprintf('Clustered PDWs, Run Number=
5 %d',irun),'FontSize',12,'FontWeight','bold');
xlabel('Pulsewidth (usec)');
ylabel('RF Frequency (MHz)');
M(irun) = getframe;

```

Table 3, step3.m, lists the program to perform ISODATA Step 3.

10 Table 3. Program STEP3.m

```

% ***** Step 3 (discard subsets with fewer than theta_n members) *****
% -----
disp('***** Step 3 *****')
n_new=0;
15 for idis=1:nc
    if n_total(idis) >= theta_n
        n_new=n_new+1;
        n_total(n_new)=n_total(idis);
    end
    % for if at line 7
20 end
    % for idis at line 6
    nc=n_new;

```

Table 4, step4.m, lists the program to perform ISODATA Step 4.

Table 4. Program STEP4.m

```

***** Step 4 (update cluster centers as their samples' means) *****
% -----
5  disp('***** Step 4 *****')
   for idis=1:nc
       if n_total(idis) ~= 0
           z(idis,:)=0;
           for kdis=1:np
10          if i_sort(kdis) == idis
               z(idis,:)=z(idis,:)+ss(kdis,:);
               end                               % for if at line 9
           end                                   % for kdis at line 8
               z(idis,:)=z(idis,:)/n_total(idis);
15          end                               % for if at line 6
       end                                     % for idis at line 5
       %
       % print new cluster centers
       %
20  disp('***** New Cluster Points *****')
       %
       %for iprint=1:nc
       %  iprint, z(iprint,:)
       %end                                     % for iprint at line 21
25  disp(z)
       %

```

Table 5, step5.m, lists the program to perform ISODATA Step 5.

Table 5. Program STEP5.m

```

% ***** Step 5 (compute average distance to cluster center within each
% cluster) *****
5  % -----
%
disp('***** Step 5 *****')
for idis=1:nc
    if n_total(idis) > 0
10     dis_c(idis)=0;
        for kdis=1:np
            if i_sort(kdis) == idis
                dis2=w1*(ss(kdis,1)-z(idis,1))^2 + w2*(ss(kdis,2)-z(idis,2))^2 + ...
                    w3*(ss(kdis,3)-z(idis,3))^2 + w4*(ss(kdis,4)-z(idis,4))^2 + ...
15                 w5*(ss(kdis,5)-z(idis,5))^2 + w6*(ss(kdis,6)-z(idis,6))^2;
                dis_c(idis)=dis_c(idis)+sqrt(dis2);
            end
            % for if at line 10
        end
        % for kdis at line 9
        dis_c(idis)=dis_c(idis)/n_total(idis);
20     end
        % for if at line 7
    end
    % for idis at line 6
    %
    % print average distance within each cluster
    %
25     disp('***** Ave Distance within each Cluster *****')
    %
    %for ipr=1:nc
    %    ipr, dis_c(ipr)
    %end
30     disp(dis_c)
    %

```

Table 6, step6.m, lists the program to perform ISODATA Step 6.

Table 6. Program STEP6.m

```

% -----
% ***** Step 6 (compute overall average distance) *****
5  % -----
%
%
disp('***** Step 6 ****')
dis_t=0;
for idis=1:nc
10
    if n_total(idis) > 0
        dis_t=dis_t + n_total(idis)*dis_c(idis);
    end
    % for if at line 8
end
    % for idis at line 7
15
%
% print overall average distance
%
disp('***** Overall Ave Distance from Clusters *****')
%
20
dis_ave=dis_t/np;
disp(dis_ave)
%
```

(Table 7 is skipped for convenience of numbering the tables so that they correspond to the numbered IDODATA steps.)

Table 8, step8.m, lists the program to perform ISODATA Step 8.

Table 8. Program STEP8.m

```

%
% -----
5  % ***** Step 8 (find STD vector for each sample set) *****
% -----
%
disp('***** At Step 7, It decides to go to Step 8 *****')
%
10  for idis=1:nc
    var1=0;
    var2=0;
    var3=0;
    var4=0;
15  var5=0;
    var6=0;
    for kdis=1:np
        if i_sort(kdis) == idis
            var1=var1+w1*(ss(kdis,1)-z(idis,1))^2;
20            var2=var2+w2*(ss(kdis,2)-z(idis,2))^2;
            var3=var3+w3*(ss(kdis,3)-z(idis,3))^2;
            var4=var4+w4*(ss(kdis,4)-z(idis,4))^2;
            var5=var5+w5*(ss(kdis,5)-z(idis,5))^2;
            var6=var6+w6*(ss(kdis,6)-z(idis,6))^2;
25            end % for if at line 16
        end % for if at line 15
        std(idis,1)= sqrt(var1/n_total(idis));
        std(idis,2)= sqrt(var2/n_total(idis));
        std(idis,3)= sqrt(var3/n_total(idis));
30        std(idis,4)= sqrt(var4/n_total(idis));
        std(idis,5)= sqrt(var5/n_total(idis));
        std(idis,6)= sqrt(var6/n_total(idis));
    end % for idis at line 8
    %
35  % print std vector for each cluster
    %
    ' ***** STD Vector for each Cluster *****'
    %
    %for ipr=1:nc
40  % ipr,std(ipr,:)
    %end % for ipr at line 37
    disp(std)
    %

```

Table 9, step9.m, lists the program to perform ISODATA Step 9.

Table 9. Program STEP9.m

```
% -----
% ***** Step 9 (find the max component of each std vector and its
5 orientation) *****
% -----
%
disp('***** Step 9 *****')
for idis=1:nc
10     stdmax(idis)=std(idis,1);
    i_stdmax(idis)=1;
end
% for idis at line 6
for idis=1:nc
    if std(idis,2) > stdmax(idis)
15         stdmax(idis)=std(idis,2);
        i_stdmax(idis)=2;
    end
    if std(idis,3) > stdmax(idis)
        stdmax(idis)=std(idis,3);
20         i_stdmax(idis)=3;
    end
    if std(idis,4) > stdmax(idis)
        stdmax(idis)=std(idis,4);
        i_stdmax(idis)=4;
25     end
    if std(idis,5) > stdmax(idis)
        stdmax(idis)=std(idis,5);
        i_stdmax(idis)=5;
    end
30     if std(idis,6) > stdmax(idis)
        stdmax(idis)=std(idis,6);
        i_stdmax(idis)=6;
    end
end
35 end
%
% print max std vector and direction of each cluster
%
disp('***** Max STD and Its Direction *****')
for ipr=1:nc
40     max_std(ipr,:) = [ipr,stdmax(ipr),i_stdmax(ipr)];
end
% for ipr at line 35
disp(max_std)
%
```

Table 10, step10.m, lists the program to perform ISODATA Step 10.

Table 10. Program STEP10.m

```

% -----
% ***** Step 10 (determine whether to split into more clusters) *****
% -----
5  %
   disp('***** Step 10 *****')
   %
   disp('***** New Splitted Clusters *****')
10  ncc=nc;
   for idis=1:nc
       if stdmax(idis) > theta_s
           if ((dis_c(idis) > dis_ave & n_total(idis) > 2*(theta_n+1)) | (nc < k/2))
               gamma=0.5*stdmax(idis);
15  split=1;
               if i_stdmax(idis) == 1
                   z(idis,1)=z(idis,1)+gamma;
                   z(ncc+1,1)=z(idis,1)-2*gamma;
                   z(ncc+1,2)=z(idis,2);
20  z(ncc+1,3)=z(idis,3);
                   z(ncc+1,4)=z(idis,4);
                   z(ncc+1,5)=z(idis,5);
                   z(ncc+1,6)=z(idis,6);
                   disp(z(idis,:))
25  disp(z(ncc+1,:))
                   ncc=ncc+1;

               elseif i_stdmax(idis) == 2
                   z(idis,2)=z(idis,2)+gamma;
30  z(ncc+1,2)=z(idis,2)-2*gamma;
                   z(ncc+1,1)=z(idis,1);
                   z(ncc+1,3)=z(idis,3);
                   z(ncc+1,4)=z(idis,4);
                   z(ncc+1,5)=z(idis,5);
35  z(ncc+1,6)=z(idis,6);
                   disp(z(idis,:))
                   disp(z(ncc+1,:))
                   ncc=ncc+1;

               elseif i_stdmax(idis) == 3
40  z(idis,3)=z(idis,3)+gamma;
                   z(ncc+1,3)=z(idis,3)-2*gamma;
                   z(ncc+1,1)=z(idis,1);
                   z(ncc+1,2)=z(idis,2);
45  z(ncc+1,4)=z(idis,4);
                   z(ncc+1,5)=z(idis,5);
                   z(ncc+1,6)=z(idis,6);

```

```

disp(z(idis,:))
disp(z(ncc+1,:))
ncc=ncc+1;

5      elseif i_stdmax(idis) == 4
        z(idis,4)=z(idis,4)+gamma;
        z(ncc+1,4)=z(idis,4)-2*gamma;
        z(ncc+1,1)=z(idis,1);
        z(ncc+1,2)=z(idis,2);
10      z(ncc+1,3)=z(idis,3);
        z(ncc+1,5)=z(idis,5);
        z(ncc+1,6)=z(idis,6);
        disp(z(idis,:))
        disp(z(ncc+1,:))
15      ncc=ncc+1;

        elseif i_stdmax(idis) == 5
        z(idis,5)=z(idis,5)+gamma;
        z(ncc+1,5)=z(idis,5)-2*gamma;
20      z(ncc+1,1)=z(idis,1);
        z(ncc+1,2)=z(idis,2);
        z(ncc+1,3)=z(idis,3);
        z(ncc+1,4)=z(idis,4);
        z(ncc+1,6)=z(idis,6);
25      disp(z(idis,:))
        disp(z(ncc+1,:))
        ncc=ncc+1;

        else
30      z(idis,6)=z(idis,6)+gamma;
        z(ncc+1,6)=z(idis,6)-2*gamma;
        z(ncc+1,1)=z(idis,1);
        z(ncc+1,2)=z(idis,2);
        z(ncc+1,3)=z(idis,3);
35      z(ncc+1,4)=z(idis,4);
        z(ncc+1,5)=z(idis,5);
        disp(z(idis,:))
        disp(z(ncc+1,:))
        ncc=ncc+1;
40

        end
        end
        end
        end
        % for if at line 14
        % for if at line 12
        % for if at line 11
        % for idis at line 10

```



Table 11, step11.m, lists the program to perform ISODATA Step 11.

Table 11. Program STEP11.m

```

% -----
% ***** Step 11 (compute pairwise distance between cluster centers and whether
5 % ***** to lump) *****
% -----
%
disp('***** Step 11 *****')
for idis1=1:nc-1
10   for idis2=2:nc
      if idis2 > idis1
          diss=w1*(z(idis1,1)-z(idis2,1))^2 + w2*(z(idis1,2)-z(idis2,2))^2 + ...
              w3*(z(idis1,3)-z(idis2,3))^2 + w4*(z(idis1,4)-z(idis2,4))^2 + ...
              w5*(z(idis1,5)-z(idis2,5))^2 + w6*(z(idis1,6)-z(idis2,6))^2;
15   dis_bw(idis1,idis2)=sqrt(diss);
      lump(idis1,idis2)=0;
      if dis_bw(idis1,idis2) <= theta_c
          lump(idis1,idis2)=1;
      end
20   end
      end
end
end
% for if at line 15
% for if at line 9
% for idis2 at line 8
% for idis1 at line 7

```

Table 12, step12.m, lists the program to perform ISODATA Step 12 and Step 13.

Table 12. Program STEP12.m

```

% -----
5 % ***** Step 12 & 13 (lump L or less clusters as allowed and updating new
% ***** cluster centers) *****
% -----
%
disp('***** Step 12 and 13 *****')
10 for idis1=1:nc-1
    for idis2=2:nc
        if (idis1 ~= idis2 & idis2 >= idis1 & lump(idis1, idis2) == 1)
            %
            z1_n = n_total(idis1)*z(idis1,1) + n_total(idis2)*z(idis2,1);
15 z(idis1,1) = z1_n / (n_total(idis1) + n_total(idis2));
            %
            z2_n = n_total(idis1)*z(idis1,2) + n_total(idis2)*z(idis2,2);
            z(idis1,2) = z2_n / (n_total(idis1) + n_total(idis2));
            %
20 z3_n = n_total(idis1)*z(idis1,3) + n_total(idis2)*z(idis2,3);
            z(idis1,3) = z3_n / (n_total(idis1) + n_total(idis2));
            %
            z4_n = n_total(idis1)*z(idis1,4) + n_total(idis2)*z(idis2,4);
            z(idis1,4) = z4_n / (n_total(idis1) + n_total(idis2));
25 z5_n = n_total(idis1)*z(idis1,5) + n_total(idis2)*z(idis2,5);
            z(idis1,5) = z5_n / (n_total(idis1) + n_total(idis2));
            %
            z6_n = n_total(idis1)*z(idis1,6) + n_total(idis2)*z(idis2,6);
30 z(idis1,6) = z6_n / (n_total(idis1) + n_total(idis2));
            %
            disp(z(idis1,:))
            z(idis2,:) = 99999;
        end
35 end
end
% for if at line 9
% for idis2 at line 8
% for idis1 at line 7

```

To illustrate the operation of the invention, a simple test case of an electronic warfare (EW) scenario consisting of five (5) radar threats was provided to

a simulation of system 20. The five radar threats and their characteristics are listed in Table 13.

Table 13. Radar Threat Characteristics for ISODATA Simulation

<u>Threat</u>	<u>RF Freq</u>	<u>PRI</u>	<u>Pulse Width</u>	<u>Pulse Amplitude</u>
1	800 - 1,000 MHz	1120 us	7 - 9 us	0 to -10 dBm
2	1000 - 1100 MHz	1820 us	13 - 15 us	-10 to -20 dBm
3	1075 - 1225 MHz	1920 us	3 -5 us	-15 to -25 dBm
4	1215 - 1405 MHz	1320 us	5.5 - 7.5 us	-30 to -40 dBm
5	1207.5 - 1212.5 MHz	2920 us	11 -13 us	-40 to -50 dBm

5                      Snapshots of PDWs of this EW scenario were generated by the program threat generator listed in Table A. FIG. 3 illustrates a snapshot of the radars' RF frequency (RF) versus TOA. FIG. 4 illustrates a snapshot of the radars' pulse width (PW) versus TOA. FIG. 5 illustrates a snapshot of the radars' pulse amplitude (PA) versus TOA. These figures indicate that radar pulses from multiple threats are  
 10 overlapped in RF, PW and PA. There, indeed, are five (5) radar threats in the input snapshots, when the snapshots are shown plotted as RF versus PW, as illustrated in FIG. 6.

Performance of system 20 in clustering and classifying the five radar threats is summarized in FIGS. 7-10. These figures illustrate the result and progress  
 15 of clustering after 1, 4, 6 and 7 iterations, respectively. The system performs well

and after seven iterations, the system clusters the input snapshots into five radar threats, without errors, as shown in FIG. 8.

It will be appreciated that system 20 may be used to cluster EW scenarios consisting of mixes of stable radars and advanced radars, such as dwell  
5 switched and frequency agile radars.

It will also be appreciated that to cluster advanced emitters having frequency agility capability, the weighs ( $w_1$ ,  $w_2$ , and others, if necessary) used in Euclidean distance calculations between PDWs may be made adaptive, so that PDWs from different threats may be sorted into different clusters and PDWs from the same  
10 threat will not be partitioned into multiple clusters. As an example, the weights may be made a function of the operational frequency band of the radar emitter and the size of clusters generated may be adjusted to prevent threat splitting.

Although illustrated and described herein with reference to certain specific embodiments, the present invention is nevertheless not intended to be  
15 limited to the details shown. Rather, various modifications may be made in the details within the scope and range of equivalents of the claims and without departing from the spirit of the invention.